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AN ANALYTICAL MODEL FOR FORECASTING NAVY OFFICER CAREER PATHS

by

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AN ANALYTICAL MODEL FOR FORECASTING NAVY OFFICER CAREER PATHS

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Abstract

A mathematical model, using Semi-Markov Processes, is suggested for the career paths of military officers. The model consists of a rectangular array of nodes with rows representing activity types and columns representing successive tours. Thus each billet of an officer is described by an activity type and a tour number. The duration of such a billet, called a tourlength, is considered fixed, given the activity type and the tour number. Transitions from activities in one tour to activities in the next tour are governed by distinct transition probability matrices. A formula is developed for the expected number of personnel occupying any activity in any tour some time in the future, given the current numbers of occupants (incumbents) in all billet types and the planned numbers of accessions in the future. Finally, an example is presented that is applicable to the Surface Warfare Officer Community of the Navy.

Keywords: Career Paths, Semi-Markov Processes

1. INTRODUCTION

Because officers of the Armed Forces experience more rigid career patterns during their professional lives than the typical career of civilians in most professional occupations, it is to be expected that careers of military officers should more readily be "modelable" in the O.R. sense. It appears that this is especially true of Naval officers who are usually expected to follow a quite stiff sequence of sea-shore rotation during their career.

In this paper an attempt is made to model analytically the career patterns of a particular class of Naval officers, namely those who belong to the Surface Warfare Officer (SWO) community, although, except for the example that will be discussed, the entire discussion is at a sufficiently general level to make the model, or at least its basic ideas, applicable to

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other officer communities of the Navy (e.g. Aviation or Submarine Warfare) and perhaps to a lesser extent of the Marine Corps, Army and Air Force as well.

2. BACKGROUND

Over the past several years a number of NPS theses have been concerned with analyzing the career paths of various parts of the Navy Officer community. Thus, Ferree [4] examined the sea tour sequence during the career of SWO's and built a computer model to compute sea tour opportunities in their community. Later, Howe [5], Ballew [2] and Poland [8] analyzed the career paths of officers in the SWO, Aviation and Nurse Corps communities, respectively, each author constructing network representations to model the alternative tour sequences during the career of an officer in each of the mentioned communities.

This theme was later picked up by Amirault [1] who developed a computer model using the network representations of SWO career paths suggested by Howe. The computer model written in Turbo Pascal for either an IBM PC or a Zenith 100 personal computer enables a user to carry out computations that simulate what happens to a corps of SWO's as they advance during their career through a succession of various types of sea and shore tours until they reach the rank of Captain or attrite from their community. This computer model was subsequently used by Mygas [7] to analyze alternative career paths in the SWO community and evaluate their advantages and disadvantages.

More recently, Steward [10] also used Amirault's model (slightly modified) to analyze the impact of the creation of the joint service officer on the career path of SWO's.

It was probably through the use of Amirault's model for various purposes that it became clear that many or most features of such a model may best be described analytically which in turn may facilitate the building of a more efficient and more useful model for personnel managers who want to experiment with various proposed changes in the career paths of (Naval) officers. It is with that hope and eventual purpose in mind that the analytical model described in this report is proposed.

3. DEFINITIONS AND NOTATIONS

<u>Tours</u>: Officers of the Armed Forces generally experience a succession of tours of various duration. For example, a SWO will begin his career (after being designated a SWO) with his Division Officer Tour (a sea tour) lasting about $2\frac{1}{2}$ years followed usually by a shore tour (which may be a tour at NPS) of about 2 year duration, and so forth. The model constructed by Amirault used twelve such tours, the last of which may be either a sea or a shore tour of about 2 year duration. At that point, a SWO is of the rank of Captain and usually has a major command. Mathematically, it will be assumed that the tour lengths are random variables, denoted by $T_1, T_2, \ldots, T_R$, where R is the total number of tours to be considered in the model.

<u>Activities</u>: Officers usually perform a great variety of tasks during their career as they advance from one tour to the next. This is probably more true in the Navy than in the other services, because of the Navy's "generalists" point of view. Nowhere is this more apparent than in the SWO community, members of which are especially expected to be "jacks of all trades". However, it is usually true that once assigned a certain job, a SWO has fairly well circumscribed tasks to perform for the duration of that tour. Thus, it is possible to delineate certain activities which are distinct from each other, even though they are an aggregation of several types of tasks in themselves. As an example, the simplest way to split the different types of tours a SWO undergoes, would be to classify all tours as either sea or shore tours. For purposes of most analyses, of course, this classification is much too oversimplified, because it does not distinguish between the activities of an officer attending a SWO Department Head course and those of another stationed, e.g. in a Pentagon or an overseas shore billet. Howe [5] selected seven activity types that he found sufficiently rich in detail for his analysis. This number was used by Amirault [1] in his model and was later found to be more or less adequate by Mygas [7] and Steward [10] for their respective analyses, although the latter was forced to redefine the seven activities for the

purpose of his investigation. Also, beginning with Howe each of the above authors found it useful to have an eighth "activity", namely "Separation" from the system also considered. This includes not only separation from the Navy, but any transfer of an officer out of the SWO community, as well as advancement beyond the twelfth tour, e.g. promotion to flag rank. Mathematically, it will be assumed here that there are A different mutually exclusive activities indexed by the letters i or j. The last of these activities may be "Separation from the System". For examples, see the authors mentioned above.

__Billets__: It is assumed that an officer is assigned a distinct billet (a job or position) during each of his/her tours of duty. This billet is always assumed to be within one of the activities. Thus, it will be said that an officer occupies a B_{jn} billet if during his/her n^{th} tour he/she is in an activity of type j. This way, an officer's career path may be described by the sequence of billets $B_{i_1 1}, B_{i_2 2}, \ldots, B_{i_R R}$, where $i_1, i_2, \ldots, i_R$ are the activity types in which the officer is engaged during his/her $1^{st}, 2^{nd}, \ldots, R^{th}$ tour. It is the transition of advancements through this sequence of billets that is the main focus of this report.

Define, for $n = 1, \ldots, R$

$$X_n = \text{the activity type occupied by an officer during his/her } n^{th} \text{ tour.}$$

Thus, "$X_n = j$" means that the officer is occupying a B_{jn} billet during his/her n^{th} tour.

It will be assumed that the activity occupied by an officer during his/her n^{th} tour depends on the activity he/she occupied on the immediately previous tour, but given that, it does not depend on activities occupied prior to the previous tour. That is precisely the assumption needed to conclude that $\{X_n, \; n = 1, 2, \ldots, R\}$ is a Markov Chain. While this assumption is not strictly valid, it is probably a quite adequate approximation of reality. The usual simplifying assumption of stationary transition probabilities, however, will be avoided.

__Time__: It would be the simplest thing to analyze the Markov Chain $\{X_n, n = 1, \ldots, R\}$

by itself which, of course, amounts to discretizing time into tour lengths. This, however, is operationally impractical as tour lengths are neither fixed nor equal in size. Instead, personnel managers are interested in analyzing their system either at any point in time (i.e. continuously) or at least at fixed and rather frequent intervals, such as monthly or quarterly. Therefore, for purposes of this report, time, designated by t, will first be assumed to be continuous and later, for computational reasons, to be measured in months or quarters.

Define, for $t \geq 0$,

$$X(t) = \text{the activity type occupied by an officer at time } t.$$

Thus, "$X(t) = j$" means the officer is occupying a type j activity at time t, i.e. is in a B_{jn} billet for some $n = 1, 2, \ldots, R$.

One of the first orders of business will be to establish the relationships between the Markov Chain $\{X_n, \ n = 1, 2, \ldots\}$ defined earlier and the Stochastic Process $\{X(t), \ t \geq 0\}$ as defined above.

4. DEVELOPMENT OF THE ANALYTICAL MODEL

In order to develop the probabilistic relationship among the variables introduced in the previous section it will be useful to define the Stochastic Process $\{X(t), \ t \geq 0\}$. This will be done very much in line with standard Renewal Theory practice even though the present model does *not* lead to a Renewal Process.

This is so, due to the fact that the sequence of tour lengths, $T_1, \ldots, T_R$ cannot be assumed to form a sequence of independent, identically distributed random variables if the actual reality of tour lengths is taken into account. A more reasonable assumption to be made is that the length of a tour depends on both the tour number (n) and on the activity (j) in which it occurs. Further, once the tour number (n) and the activity (j) are both specified the tour length is essentially fixed, with very little chance of variation. Thus, e.g. an officer assigned to a Professional Education type of activity on his/her 2^{nd} tour will serve

there 2 years, whereas if he/she is assigned instead to another tour in the Fleet, his/her tour length will be $1\frac{1}{2}$ years.

Therefore, the mathematical assumption to be made is that, for $n = 1, \ldots, R$ and $j = 1, \ldots, A$,

$$T_n = l_n(j) \ \text{ if } \ X_n = j \tag{1}$$

where $l_n(j)$ is a fixed duration of time depending on j and n only.

The Markov Chain $\{X_n, \ n = 1, 2, \ldots, R\}$ is assumed to be governed by non-stationary transition probabilities

$$P_{ij}(n) = P\,(X_{n+1} = j \mid X_n = i)$$

for $i, j = 1, \ldots, A$ and $n = 1, \ldots, R-1$.

Unfortunately, an assumption of stationarity would *not* be realistic as the probability of transiting from activity i to activity j will depend heavily on the tour number (n) at which the officer does the transiting.

It should be quite clear now that the random variables $T_1, \ldots, T_R$ are dependent, since e.g. T_{n+1} depends on X_{n+1} which in turn depends on X_n, and so T_{n+1} and T_n are also dependent. This, of course, invalidates the results of Renewal Theory, but nonetheless some of the concepts and ideas can be salvaged.

Thus, the following variables and processes are introduced. Let

$$S_0 = 0 \quad \text{and} \quad S_n = T_1 + \ldots + T_n \quad \text{for} \quad n = 1, \ldots, R.$$

Also, define for $t \geq 0$,

$$N(t) = \text{the number of tours completed by an officer during the period } [0, t).$$

Note that "$N(t) = n$" means that the officer has *completed* exactly n tours by time t and must consequently be in his/her $(n+1)^{st}$ tour. To make this precise even when the n^{th} tour is completed at *exactly* time t (as will occasionally be the case with discrete tour lengths) the convention will be adopted that at the moment of completion of a tour the

officer "is considered to be in his/her next tour already". Of course, in practice there is always a lag time (of days or weeks) necessary for an officer to make a so-called "permanent change of station" move. This, however, is both short compared to tour lengths, as well as unimportant in the complete career structure of an officer.

From a mathematical point of view a tour completion could be regarded as a "renewal" in which case $\{N(t),\ t \geq 0\}$ would be regarded as a renewal counting process (see e.g. Ross [9]). Unfortunately, due to the dependence among the random variables, $T_1, \ldots, T_R$, the terminology "renewal" is inappropriate.

Even though $\{N(t),\ t \geq 0\}$ is *not* a renewal counting process, some of the basic relations of Renewal Theory are still valid. In particular,

$$N(t) = n \quad \text{iff} \quad S_n \leq t < S_{n+1} \tag{2}$$

is still valid for $n = 0, 1, \ldots, R - 1$.

The reason for the relationship is the same as in standard Renewal Theory (see e.g. Mode [6], Ross [9] or Feller [3]). Namely, if the officer has completed his/her n^{th} tour by time t, but not his/her $(n + 1)^{st}$ tour then t must exceed or equal the total time the officer has spent in his/her first n tours, but not the total time spent in his/her first $n + 1$ tours. Likewise, the converse statement is also valid as usual.

It is not the purpose of this report to explore the extent to which the results of Renewal Theory may be salvaged in this model. For one thing, the most elegant results of Renewal Theory are asymptotic, i.e. apply for large t and n. Here, such results would be of little practical value, since the largest value of n is R which may be around 12 or 15, at most. More importantly, however, even if R had a large enough value to make asymptotic results relevant, from a practical point of view it is of more interest to know what happens for small values of n and t, where the population of officers is much larger than for big values of n and t where there are relatively fewer officers. For that reason, the rest of the development favors an approach that leads to computationally useful results.

From a practical point of view it is important to know the number of officers serving in

billets B_{jn} at any particular time t, for all possible values of j and n. With that goal in mind, the probability that an officer serves in a billet B_{jn+1} at time t is sought first. This probability may be expressed as

$$H_{jn+1}(t) = P(X(t) = j,\ N(t) = n) = P(S_n \le t < S_{n+1},\ X_{n+1} = j) \tag{3}$$

for $t \geq 0$, $n = 0, 1, \ldots, R-1$ and $j = 1, \ldots, A$. This equality follows from the relationship (2) above, and the fact that an officer may serve in a B_{jn+1} billet at time t iff his/her $(n+1)^{st}$ tour is in activity j and $S_n \le t < S_{n+1}$. Another way to express this fact is to state that for $t \geq 0$ and $n = 0, 1, \ldots, R-1$,

$$X(t) = X_{n+1} \quad \text{if} \quad S_n \le t < S_{n+1} \tag{4}$$

This point of view shows that $\{X(t),\ t \geq 0\}$ is a Semi-Markov Process (see e.g. Mode[6] or Ross[9]) with non-stationary transition probabilities and sojourn times (tour lengths) dependent on the state (activity) which the process enters at time n as well as on n.

The probability on the right hand side of equation (3) may be evaluated without too much difficulty. To start with the easiest case, let $n = 0$. Then, for $t \geq 0$,

$$\begin{aligned} P(S_0 \le t < S_1,\ X_1 = j) &= P(T_1 > t,\ X_1 = j) \\ &= P(l_1(j) > t,\ X_1 = j) = P(l_1(j) > t)\pi_j \end{aligned}$$

because $\pi_j = P(X_1 = j)$ must be given for all $j = 1, \ldots, A$ and $T_1 = l_1(j)$ when $X_1 = j$ with probability one. Therefore, the $n = 0$ case results in

$$\begin{aligned} H_{j1}(t) = P(X(t) = j,\ N(t) = 0) &= P(S_0 \le t < S_1,\ X_1 = j) \\ &= \pi_j I(t;\ 0,\ l_1(j)) \end{aligned}$$

where the indicator function of the interval $[a, b)$ is defined as

$$I(t;\ a,\ b) = \begin{cases} 1 & \text{if } a \le t < b \\ 0 & \text{otherwise} \end{cases}.$$

The next case, $n = 1$, is solved similarly:

$$\begin{aligned} H_{j2}(t) &= P(X(t) = j,\ N(t) = 1) = P(S_1 \le t < S_2,\ X_2 = j) \\ &= \sum_{i=1}^{A} P(S_1 \le t < S_2,\ X_2 = j \mid X_1 = i)\pi_i \end{aligned}$$

But

$$P(S_1 \le t < S_2,\ X_2 = j \mid X_1 = i) = P(l_1(i) \le t < l_1(i) + l_2(j),\ \ X_2 = j \mid X_1 = i)$$

$$= P_{ij}(1) I(t;\ l_1(i),\ l_1(i) + l_2(j))$$

because when $X_1 = i$ and $X_2 = j$,

$$S_1 = T_1 = l_1(i) \ \text{ and } \ S_2 = T_1 + T_2 = l_1(i) + l_2(j).$$

Therefore,

$$H_{j2}(t) = \sum_{i=1}^{A} \pi_i P_{ij}(1) I(t;\ l_1(i),\ l_1(i) + l_2(j)).$$

The general case is treated likewise:

$$H_{j,n+1}(t) = P(X(t) = j,\ N(t) = n) = P(S_n \le t < S_{n+1},\ X_{n+1} = j)$$

$$= \sum_{i_1,\ldots,i_n=1}^{A} P(S_n \le t < S_{n+1},\ X_{n+1} = j \mid X_1 = i_1, \ldots, X_n = i_n)$$

$$\times P(X_1 = i_1, \ldots, X_n = i_n)$$

where the summation is an n-tuple summation over the indices $i_1, \ldots, i_n$, each extended over the values 1 through A.

Then,

$$P(X_1 = i_1, \ldots,\ X_n = i_n) = \pi_{i_1} P_{i_1 i_2}(1) \ldots P_{i_{n-1} i_n}(n-1)$$

using the standard Markov Chain argument (see e.g. Ross [8]). Also, by way of the Markov property,

$$P(S_n \le t < S_{n+1},\ X_{n+1} = j \mid X_1 = i_1, \ldots, X_n = i_n)$$

$$= P(l_1(i_1) + \ldots + l_n(i_n) \le t < l_1(i_1) + \ldots + l_n(i_n) + l_{n+1}(j),\ X_{n+1} = j \mid X_n = i_n)$$

$$= P_{i_n j}(n)\ \ I(t;\ l_1(i_1) + \ldots + l_n(i_n), l_1(i_1) + \ldots + l_n(i_n) + l_{n+1}(j))$$

To ease the notational complexity the following abbreviated notation is introduced:

$$s_n = s_n(i_1, \ldots, i_n) = l_1(i_1) + \ldots + l_n(i_n)$$

and

$$s_{n+1} = s_{n+1}(i_1, \ldots, i_n, j) = l_1(i_1) + \ldots + l_n(i_n) + l_{n+1}(j)$$

Therefore, the formula derived is, for $j = 1, \ldots, A$ and $n = 0, 1, \ldots, R-1$,

$$H_{jn+1}(t) = \sum_{i_1, \ldots, i_n = 1}^{A} \pi_{i_1} P_{i_1 i_2}(1) \ldots P_{i_{n-1} i_n}(n-1) P_{i_n j}(n) I(t;\ s_n,\ s_{n+1}) \tag{5}$$

A numerical example is presented in the next section.

Formula (5) may be used to compute the expected number of officers in billet B_{jn+1} at time t, provided at time zero *all* officers start out in the first tour billets B_{i1}, $i = 1, \ldots, A$ and all other billets B_{jn} for $n = 2, \ldots, R$ and $j = 1, \ldots, A$ are empty at that time. In practice, however, this is hardly the case, since billets in all tours are occupied at all times. To accommodate this empirical fact Formula (5) will first be specialized slightly, then reassigned a somewhat more complex notation and finally extended to the case where an officer starts out, at time zero, in a tour later than the first tour and/or has already served some time in the billet he/she is occupying at time zero.

Formula (5) is first specialized to the case when an officer starts out in a billet B_{i1} with probability one, i.e.

$$\pi_j = \begin{cases} 1 & \text{if } j = i \\ 0 & \text{otherwise.} \end{cases}$$

In actual practice *all* SWO's start out in a Fleet Unit Activity billet as Division Officers (see example in next section), but the specialization is useful for the theoretical development of the model as well. Thus Formula (5) can be rewritten as

$$\begin{aligned} H^{*}_{jn+1}(t) &= P(X(t) = j,\ N(t) = n \mid X(0) = i,\ N(0) = 0,\ T_1 = l_1(i)) \\ &= \sum_{i_2, \ldots, i_n = 1}^{A} P_{i i_2}(1) \ldots P_{i_{n-1} i_n}(n-1) P_{i_n j}(n) I(t;\ s_n,\ s_{n+1}) \end{aligned} \tag{6}$$

where the conditions indicate that the officer started out in a billet B_{i1} and has not served any time in that billet prior to time zero. This latter fact is shown by the condition, $T_1 = l_1(i)$, that he/she has the entire tour-length, $l_1(i)$, to serve in billet B_{i1}. This fact will be subsequently referred to as the officer having no "prior experience" in that billet.

If, on the other hand, the officer has served a specific time period, say τ, prior to time zero, in billet B_{i1}, then he/she has only $l_1(i) - \tau$ to serve in that billet. This will be denoted

by

$$H^{i}_{jn+1}(t;\ \tau) = P(X(t) = j,\ N(t) = n \mid X(0) = i,\ N(0) = 0,\ T_1 = l_1(i) - \tau) \qquad (7)$$

and Formula (6) is easily altered by simply replacing $l_1(i)$ by $l_1(i) - \tau$.

Finally, Formula (6) may be generalized to the case where the officer starts out in a billet B_{im+1} with prior experience τ in that billet at time zero:

$$H^{im+1}_{jn+1}(t;\ \tau) = P(X(t) = j,\ N(t) = n \mid X(0) = i,\ N(0) = m,\ T_{m+1} = l_{m+1}(i) - \tau)$$

for $m = 0, 1, \ldots, n$.

Formula (6) may easily be altered to fit $H^{im+1}_{jn+1}(t;\ \tau)$ as well. First the tour-length $l_{m+1}(i)$ is replaced by $l_{m+1}(i) - \tau$. Then the product $P_{ii_2}(1)\ldots P_{i_{n-1}i_n}(n-1)P_{i_nj}(n)$ is replaced by the shorter product

$$P_{ii_{m+2}}(m+1)\ldots P_{i_{n-1}i_n}(n-1)P_{i_nj}(n)$$

and the $(n-1)$-tuple summation of Formula (6) is likewise replaced by a shorter $(n-m-1)$-tuple summation. Thus,

$$H^{im+1}_{jn+1}\ (t;\ \tau) = \sum_{i_{m+2},\ldots,i_n=1}^{A} P_{ii_{m+2}}(m+1),\ldots,P_{i_{n-1}i_n}(n-1)P_{i_nj}(n)I(t;\ s^*_{n-m}, s^*_{n-m+1}) \qquad (8)$$

where s^*_{n-m} and s^*_{n-m+1} are sums of tour-lengths, namely,

$$s^*_{n-m} = l_{m+1}(i) - \tau + l_{m+2}(i_{m+2}) + \ldots + l_n(i_n)$$
$$s^*_{n-m+1} = s^*_{n-m} + l_{n+1}(j).$$

Formula (8) is valid for

$$i, j = 1, \ldots, A;\ \ m = 0, 1, \ldots, R-1;\ \ n = m+2, \ldots, R-1;\ \ 0 \le \tau < l_{m+1}(i);\ \ t \ge 0.$$

The $n = m$ and $n = m + 1$ cases are covered respectively by the formulas

$$H^{im+1}_{jm+1}(t;\ \tau) = \delta_{ij} I(t;\ 0,\ l_{m+1}(i) - \tau),$$

where $\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$

and

$$H^{im+1}_{jm+2}(t;\ \tau) = P_{ij}(m+1)I(t;\ l_{m+1}(i) - \tau, l_{m+1}(i) - \tau + l_{m+2}(j))$$

for $i, j = 1, \ldots, A;\ \ m = 0, 1, \ldots, R-1;\ \ 0 \leq \tau < l_{m+1}(i);\ \ t \geq 0.$

For practical use of this model an analyst may wish to observe the current state of affairs, i.e. note the number of officers currently occupying billets B_{im+1}, together with their accumulated experience τ, in that billet, and then forecast the number of officers who will occupy billets B_{jn+1} at some future time t. With this purpose in mind the following notation is introduced:

$Q_{jn}(t;\ \tau) =$ number of officers occupying billets B_{jn} with prior experience τ in that billet at time t

and

$$Q_{jn}(t) = \sum_{\tau=0}^{l_n(j)-1} Q_{jn}(t;\ \tau)$$

$=$ total number of officers occupying billets B_{jn} at time t.

Note that by summing over τ the assumption has been made that time is observed in discrete increments of some unit that will be taken to be one month in the example of the next section. Alternatively, the summation could be replaced by an integral if τ is thought be a continuous quantity.

The formula for the expected number of officers occupying billets B_{jn+1} at time t may be obtained under the assumption that the number of officers occupying billets B_{im+1}, with prior experience τ, at time zero are known for all possible values of τ, i and m.

In fact, assume that the values of

$Q_{im+1}(0;\ \tau) =$ number of officers occupying billets B_{im+1} with prior experience τ at time zero

are given for all $\tau = 0, 1, \ldots, l_{m+1}(i) - 1;\ \ i = 1, \ldots, A$ and $m = 0, 1, \ldots, R-1$.

Finally, it must be assumed that the number of new officers entering the system from time to time is also known. Here, it will be assumed that new officers enter the system every

unit of time, but only in Tour 1. These new officers may be thought of as having "negative experience" at time zero in billet B_{i1} and consequently the following notation may be used.

For $i = 1, \ldots, A$ and $\tau = 1, \ldots, t$ let

$$Q_{i1}(0;\ -\tau) = \text{number of new officers entering billet } B_{i1} \text{ at time } \tau.$$

Then, the expected number of officers in billets B_{jn+1} at time t is

$$\begin{aligned} E[Q_{jn+1}(t)] &= \sum_{m=0}^{R-1} \sum_{i=1}^{A} \sum_{\tau=0}^{l_{m+1}(i)-1} Q_{im+1}(0;\ \tau) H_{j\ n+1}^{i\ m+1}(t;\ \tau) \\ &+ \sum_{i=1}^{A} \sum_{\tau=1}^{t} Q_{i1}(0;\ -\tau) H_{jn+1}^{i1}(t;\ -\tau) \end{aligned} \tag{9}$$

for $j = 1, \ldots, A$; $n = 0, 1, \ldots, R-1$ and $t \geq 0$.

Note that the notation $H_{jn+1}^{i1}(t;\ -\tau)$ using "negative experience", $-\tau$, in billets B_{i1} is still defined as in Formula (7), i.e. provided $\tau = 1, \ldots, t$,

$$H_{jn+1}^{i1}(t;\ -\tau) = P(X(t) = j, N(t) = n \mid X(0) = i, N(0) = 0, T_1 = l_1(i) + \tau)$$

and Formula (8) is correct without change.

In the example of the next section $Q_{i1}(0;\ -\tau) = C_i$ for all $\tau = 1, \ldots, t$ and, the second term in Formula (9) simplifies to

$$\sum_{i=1}^{A} C_i \sum_{\tau=1}^{t} H_{jn+1}^{i1}(t;\ -\tau).$$

5. A Numerical Example

One possible utilization of this model is to analyze the career paths of SWO's for the purpose of determining how newly created joint tours with the other military services may best be integrated into the traditional career structure of SWO's. Following Steward [10] the activities in which SWO's are engaged have been divided into seven categories:

1. Professional Training
2. Professional Education

3. Joint Education

4. Joint Tour

5. Fleet Unit

6. Afloat Staff

7. Shore (other than those listed above).

The precise definition as well as the rationale for such a division is explained in Steward [10]. Suffice it to mention that the emphasis here lies on activities 3 and 4 in order to analyze the feasibility to integrate these new activities into the traditional Navy SWO career structure. An eighth activity called "Separation" (from the SWO community) is normally also included, but is omitted here, because the probabilities of "separation" are implied by the probabilities of being in the other activities and, more importantly, they are of little interest to the user in a typical analysis.

The formulas of the previous section were programmed in APL on the IBM 3033 mainframe computer at the Naval Postgraduate School. Programming presented interesting challenges that will be discussed in a subsequent report. The purpose of this section is simply to illustrate through this example the use of the formulas of the previous section.

Throughout the example it will be assumed that the seven activities are as given above and there are twelve tours to be considered, i.e. $A = 7$ and $R = 12$. The 7 x 12 matrix below provides the tourlength data, in months, assumed to be valid for each billet type B_{in}, for $i = 1, \ldots, 7$ and $n = 1, \ldots, 12$:

	Tours											
Activity	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	6	0	0	0	0	0	0	0	0	0
2	0	24	0	0	0	18	0	0	12	0	12	0
3	0	0	0	0	0	6	0	0	12	0	12	0
4	0	0	0	0	0	24	24	0	30	24	0	30
5	30	18	0	18	18	0	18	21	0	27	27	24
6	0	18	0	0	18	24	18	18	0	24	24	24
7	0	24	0	0	0	24	0	0	24	24	24	24

Note that a zero tourlength, $l_n(i) = 0$, implies that B_{in} is not a feasible billet. Several of the billet types are in this category, a fact that must be utilized in the program to assure its efficiency. Also note that all SWO's *must* start out in billets B_{51} i.e. the first tour must be in a Fleet Unit Activity.

The subsequent routing of SWO's through the network of B_{in} billets is then determined by the eleven transition matrices $P(n), n = 1, \ldots, 11$ given below:

$$\underline{P}(1) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0.25 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \underline{P}(2) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.65 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{P}(3) = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \underline{P}(4) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.76 & 0.19 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{P}(5) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0.1 & 0.1 & 0 & 0 & 0.55 \\ 0 & 0.1 & 0.1 & 0.1 & 0 & 0 & 0.55 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \underline{P}(6) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.6 & 0 \\ 0 & 0.2 & 0.45 & 0.35 & 0 & 0 & 0 \\ 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.4 & 0.58 & 0 \end{bmatrix}$$

$$\underline{P}(7) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0.48 & 0 \\ 0 & 0 & 0 & 0 & 0.98 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \underline{P}(8) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.15 & 0.1 & 0.1 & 0 & 0 & 0.6 \\ 0 & 0.15 & 0.1 & 0.1 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\underline{P}(9) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.12 & 0.88 & 0 \\ 0 & 0 & 0 & 0.2 & 0.12 & 0.68 & 0 \\ 0 & 0 & 0 & 0 & 0.12 & 0.88 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.12 & 0.38 & 0.35 \end{bmatrix} \quad \underline{P}(10) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0.05 & 0.02 & 0 & 0.1 & 0.5 & 0.3 \\ 0 & 0 & 0 & 0 & 0.12 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.05 & 0 & 0 \end{bmatrix}$$

$$\underline{P}(11) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.42 & 0.3 & 0.28 \\ 0 & 0 & 0 & 0.15 & 0.32 & 0.28 & 0.25 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0.1 & 0.3 \\ 0 & 0 & 0 & 0 & 0.5 & 0.1 & 0.3 \end{bmatrix}$$

Note that the transition matrices also reflect the fact that many billet types are infeasible accounting for the large number of zeroes among the matrix elements. Most rows do not sum to one due to the possibility of separation from the SWO community in such cases. For example, in the matrix $\underline{P}(2)$ the only non-zero element in the fifth row is 0.5 in the first column, indicating that 50% of those serving in billet B_{52} transfer to billet B_{13} at the end of their (second) tour. The other 50% apparently attrite from the community.

With the above data the probabilities that an officer who is now (at time zero) occupying a billet B_{54} and has accumulated nine months experience in that billet will be in any billet B_{jn} ninety-six months (i.e. eight years) from now may be computed using Formula (8). The result, for all $j = 1, \ldots, 7$ and $n = 1, \ldots, 12$, is given by the 7 x 12 matrix below:

Activity	Tours 1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0.082	0	0	0
3	0	0	0	0	0	0	0	0	0.055	0	0	0
4	0	0	0	0	0	0	0	0	0.068	0.003	0	0
5	0	0	0	0	0	0	0	0	0	0.004	0	0
6	0	0	0	0	0	0	0	0	0	0.025	0	0
7	0	0	0	0	0	0	0	0	0.0405	0	0	0

Clearly, these probabilities are positive for only some specific tours, in this case for Tours 9 and 10 only. Since the sum of all these probabilities is .642, the probability that such an officer attrites from the SWO community within the ninety-six month period is .358.

To use Formula (9) to project an entire force of SWO's into the future, the current force structure as well as planned future accessions must be given or assumed. The 7 x 12 matrix below gives the number of incumbents currently (at time zero) occupying billets B_{im} for $i = 1, \ldots, 7$ and $m = 1, \ldots, 12$:

Activity	Tours 1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	222	0	0	0	0	0	0	0	0	0
2	0	264	0	0	0	54	0	0	72	0	0	0
3	0	0	0	0	0	18	0	0	48	0	0	0
4	0	0	0	0	0	168	0	0	120	48	0	90
5	3240	468	0	972	468	0	198	714	0	108	135	48
6	0	198	0	0	108	0	468	90	0	384	48	0
7	0	264	0	0	0	672	0	0	552	192	24	96

However, to make use of Formula (9) the number of incumbents in each billet type B_{im} *and* with experience r for $r = 0, 1, \ldots, l_m(i) - 1$ must also be known. For the sake of simplicity it is assumed in this example that the incumbents in billet B_{im}, whose number is given above, is distributed *evenly* among all possible experience (r) values. For example, 108 of the total 3240 officers currently occupying billet type B_{51} have accumulated each of $0, 1, \ldots, 29$ months of experience in that billet. On the other hand, only 37 officers of the total 222 incumbents in billet type B_{13} have accumulated each of $0, 1, \ldots, 5$ months of

experience in that billet. The difference is due to the fact that tourlengths in various billets differ, namely $l_1(5) = 30$ months whereas $l_3(1) = 6$ months.

The number of future accessions will be assumed to be the same, say 110 officers, for each of the next twenty-five months. As mentioned earlier, all accessions in the SWO community must start out as Division Officers, i.e. in a Fleet Unit Activity. Therefore,

$$Q_{i1}(0;\ -\tau) = C_i = \begin{cases} 110 & \text{if } i = 5 \\ 0 & \text{if } i = 1,2,3,4,6,7 \end{cases}$$

for all $\tau = 1, \ldots, 25$.

With all this data Formula (9) allows calculation of the expected number of officers occupying B_{jn} billets twenty-five months from now ($t = 25$) for all $j = 1, \ldots, 7$ and $n = 1, \ldots, 12$. The results are rounded to the nearest integers, and given in the table below:

	Tours											
Activity	1	2	3	4	5	6	7	8	9	10	11	12
1	0	0	236	0	0	0	0	0	0	0	0	0
2	0	259	0	0	0	71	0	0	70	0	2	0
3	0	0	0	0	0	31	0	0	47	0	1	0
4	0	0	0	0	0	90	109	0	118	19	0	78
5	3290	486	0	669	648	0	220	659	0	119	127	38
6	0	194	0	0	162	0	320	100	0	484	48	8
7	0	778	0	0	0	497	0	0	563	193	29	83

Note, however, that the program is flexible enough to accept accessions that vary from month to month as well.

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